

LOGARITHMS [1.4]

1(a)  $3^{\log_3 w} = W$

$3^{\log_3 w} = \square$   
 $\log_3 [3^{\log_3 w}] = \log_3 \square$   
 $\log_3 w = \log_3 \square \therefore w = \square$

(b)  $\log_3(3^w) = W$

$\log_3 3^w = \square$   
 $3^\square = 3^w$   
 $\therefore \square = w$

2. If  $g(x) = \ln(ax + 2)$ , where  $a \neq 0$ , what is the effect of increasing  $a$  on  
 (a) the y-intercept? (b) the x-intercept?

$g(0) = \ln(a \cdot 0 + 2)$   
 $g(0) = \ln 2$  A CONSTANT NOT A VARIABLE  
**NO EFFECT**

$0 = \ln(ax + 2)$   
 $e^0 = ax + 2$  IF "a" IS INCREASING  
 $ax = -1$   $x = -\frac{1}{a}$  APPROACHES 0 FROM THE LEFT.

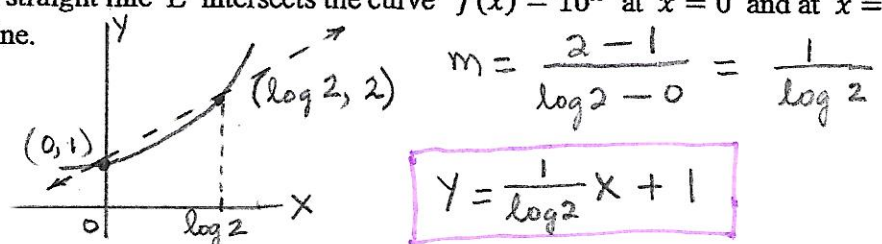
3. The size of an exponentially growing bacteria colony doubles in 7 hours. How long will it take for the number of bacteria to triple?

$Q = Q_0 e^{kt}$  TO TRIPLE  
 $2 = e^{k \cdot 7}$   $kt = \frac{1}{7} \ln 2$   $3Q_0 = Q_0 e^{kt}$   
 $\ln 2 = 7k$   $kt = \ln 2^{1/7}$   $3Q_0 = Q_0 e^{\ln(2)^{1/7}}$   
 $k = \frac{1}{7} \ln 2$   $3 = 2^{t/7}$   $\ln 3 = \frac{t}{7} \ln 2$   
 $t = \frac{7 \ln 3}{\ln 2} \approx 11.0947$  HRS.

4. Find the half-life of a radioactive substance that is reduced by 30% in 20 hours.

amount remaining =  $0.7 P_0$   
 $0.7 P_0 = P_0 (0.5)^{20/H}$   
 $\ln(0.7) = \frac{20}{H} \ln(0.5)$   
 $H = \frac{20 \ln(0.5)}{0.7} \approx 38.867$  HRS

5. A straight line L intersects the curve  $f(x) = 10^x$  at  $x = 0$  and at  $x = \log 2$ . Find the equation of the line.



6. The exponential function  $Q(m) = ke^{m\tau}$  satisfies the conditions  $Q(0) = 4e$  and  $Q(2) = 4$ . Find the constants  $k$  and  $\tau$ .

$(0, 4e)$   $(2, 4)$   
 $4e = ke^0$   $4 = 4e(e^{2\tau})$   
 $k = 4e$   $1 = e^{1+2\tau}$   
 $\ln 1 = 1 + 2\tau$   
 $0 = 1 + 2\tau$   
 $-\frac{1}{2} = \tau$